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A systematic review of augmented reality in mathematics education: Fostering learning through art integration

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Abstract

Augmented reality (AR) is acknowledged as a promising educational tool fostering the manipulation, visualization, and contextualization of abstract concepts to enhance student motivation and comprehension. However, the lack of educator training in AR implementation underscores the necessity for further research and support for effective integration into teaching practices. To this end, this article conducts a systematic review based on the PRISMA guidelines to analyze 20 English-language journal articles from the SCOPUS database, wherein geometry emerges as the most extensively studied topic with AR potential. This paper provides insights into the successful integration and impact of AR in mathematics education along with an exploration of incorporating art elements in aiding students' understanding of mathematical concepts and their social–emotional and cognitive development. Furthermore, this study examines challenges in using AR technology in mathematics education, such as teacher training and technical implementation. The findings of this study are expected to provide a clearer understanding of the potential role of AR in mathematics education.

Keywords: Augmented reality; Mathematics education; Arts in mathematics; Systematic review

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1. Introduction

Innovative approaches for enhancing mathematics education involve the integration of advanced technologies such as virtual reality (VR) and augmented reality (AR), which offer unique opportunities for the creation of interactive and engaging learning experiences. Unlike VR, which immerses users in a fully simulated environment and requires specialized equipment that limit mobility and increase cost, AR overlays digital elements such as graphics, text, and three-dimensional (3D) models onto the real world, enhancing students' interaction with their surroundings. This technology allows educators to create visual models of mathematical concepts, significantly improving students' spatial reasoning and understanding of processes, properties, and theorem proofs.[1]

Regarding the use of VR in mathematics education, the NeoTrie VR project exemplifies how VR can be used to enhance geometry education. By allowing students to interact with

3D geometric shapes in virtual environments, NeoTrie VR strengthens students' spatial reasoning and understanding of complex concepts. Moreover, its immersive nature fosters collaborative learning by making it possible for multiple users to engage simultaneously, thereby increasing student motivation and interest in geometry. Its flexibility also enables teachers to tailor lessons to various educational levels, making geometry education more interactive and effective.[2] The platform enables learners to explore and manipulate 3D objects in ways that traditional methods cannot, leading to a deeper understanding of geometric concepts. Therefore, for teachers, NeoTrie VR offers a powerful tool to make abstract geometric properties more tangible and accessible.[3]

Meanwhile, AR is also a promising tool for educators.[4] According to Azuma[5] (1993), AR merges virtual and real-world elements in real-time within a 3D space. Utilizing devices such as smartphones, tablets, headsets, and smart glasses, AR overlays computer-generated information onto users' physical surroundings. Hence, AR exhibits three core characteristics: real-time interaction, seamless integration of the digital and physical worlds, and the provision of contextual information to enhance user understanding.[4,6,7] The previous studies have confirmed AR's potential across all educational levels from early childhood to university, as well as its potential to cater to diverse student populations including those with special needs. AR offers multiple learning opportunities with numerous benefits for teaching and learning,[8] such as improving performance, increasing motivation, enabling new learning experiences, saving time, enhancing lab skills and attitudes, and fostering critical thinking, problem-solving, and communication skills.[9-11] In particular, mathematics education presents a natural fit for AR integration due to the potential benefits in manipulation, visualization, and authentic contextualization.[12] Integrating AR into mathematics education environments has been shown to enhance students' motivation, engagement, and comprehension.[4] For instance, AR-enhanced field trips to museums, architectural sites, or natural landmarks can provide students with interactive learning experiences and additional information.[13] Furthermore, incorporating game elements and challenges into mathematics instruction using AR can increase student engagement and enjoyment.[14] Combining STEM (Science, Technology, Engineering, and Mathematics) subjects with the arts (STEAM) promotes active learning, offering a more engaging education.[15]

While AR shows promise in enhancing student participation and understanding, many teachers lack training in implementing this technology. Research on AR implementation in schools is insufficient, leaving educators ill-equipped to effectively use it in classrooms. More research is needed to support teachers in effectively integrating AR into teaching practices.[16] Recent technical, infrastructural, and societal developments have therefore recognized the potential of AR in the context of mathematics education research. Scholars and educators have reported a variety of outcomes; however, the lack of a summary of these empirical studies prevents stakeholders from forming a clear view of the benefits and challenges. Therefore, this article aims to provide a review of the current research on the evolving role of AR in mathematics education. Moreover, the potential of integrating mathematics curricula with arts and culture through AR will also be explored. We aim to summarize findings, guide future studies, and reflect on major achievements in the field. In particular, we explore the following research questions:

- RQ1: In which domains of mathematics education is AR technology currently applied, what tools are used, and what are the outcomes?
- RQ2: What are the challenges of employing AR technology in teaching mathematics?
- RQ3: How can mathematics be combined with the arts, architecture, and culture through AR-supported technology, and in how is this accomplished?

2. Data and methods

2.1. Research settings

Following the Preferred Reporting Items for Systematic reviews and Meta-Analyses (PRISMA) guidelines, two researchers independently conducted a systematic peer-reviewed search in the Scopus online database and then collaborated for the final selection. Our search key terms were "mathematics" and "augmented reality." We also included the terms "education" and "arts."

According to the previous studies,[17] there has been a sudden increase in the number of publications since 2013, with publications reaching the highest level in 2016. Therefore, we explored the situation from 2017 and beyond.[17] The period under study was 7 years, from January 2017 to January 2024. The search yielded 545 papers.

The key themes and findings were analyzed to provide insights into the implementation of AR tools and the pedagogical strategies employed to integrate the arts into mathematical instruction. During the screening stage, we used five criteria for selecting manuscripts for study: 1) original articles; 2) written in English; 3) published between January 2017 and January 2024; 4) in the field of mathematics education; and 5) provide empirical results on AR based on our research questions.

Based on these criteria, we screened the titles, abstracts, and keywords of the 545 studies, which resulted in 25 studies, five of which were further excluded because the full texts of three were unavailable and two did not respond to our research questions. We then examined the full texts of the remaining 20 studies in our systematic review. The screening process was conducted manually based on the Scopus database results.

Of the studies not included, 95 were rejected because they were published outside the 2017 – 2024 period. A further 363 were rejected because they were not related to mathematics education, and 62 were rejected due to the language and type of the document (Figure 1).

2.2. Papers collected

Following our screening process, we analyzed the remaining 20 studies according to our classification scheme, which is described below.

2.2.1. Utility, application, and effectiveness of AR in mathematical education

At first, we were interested in the publication year of these papers. Of the 20 studies reviewed from 2017 through 2024, most (15) were published from 2020 and beyond (Figure 2).

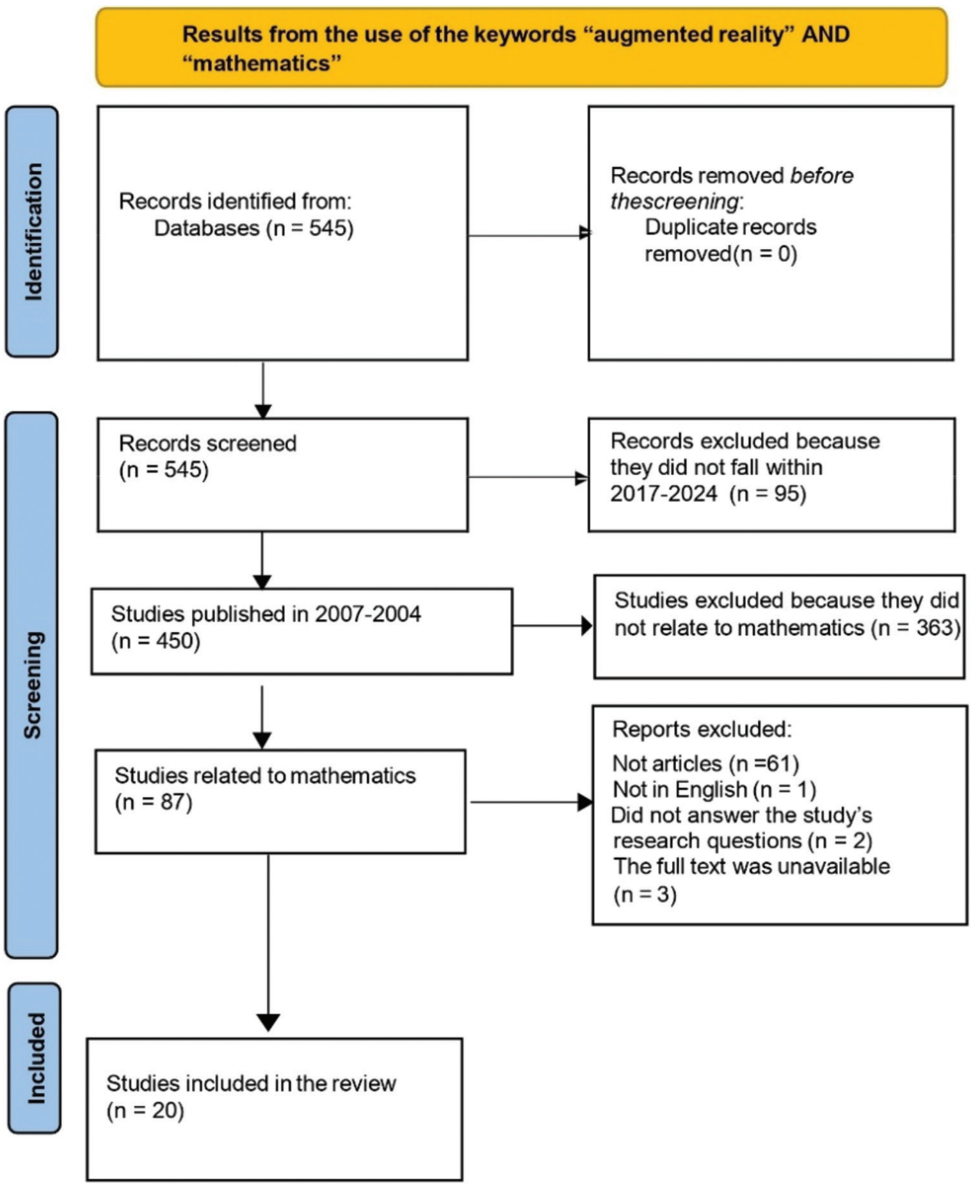

Figure 1. PRISMA flow diagram

An unexpected point of interest concerned the countries where the studies were conducted. According to our results, 12 countries published these papers (Table 1). Spain and Austria produced the highest number of articles (n = 5 and n = 4, respectively), and one article[18] was coauthored by scientists from both countries. Mexico and Saudi Arabia published two articles each. Italy also published two articles, but one was cosigned with Israel. The remaining countries only published one article each.

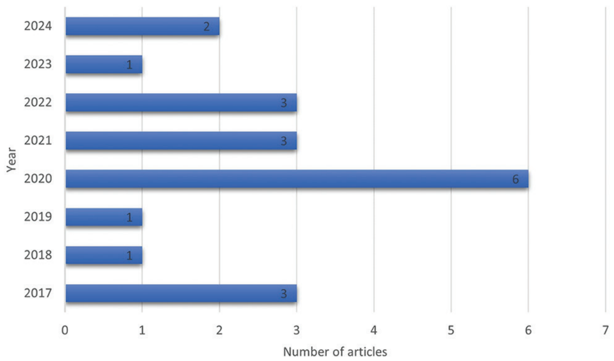

Figure 2. Number of articles by year

Table 1. List of articles by country

Country	Total	References
China	1	19
Saudi Arabia	2	20, 21
Spain	5	18, 22-25
Mexico	2	26, 27
Malaysia	1	28
Korea	1	29
Ukraine	1	30
Ecuador	1	31
Israel	1	32
Austria	4	18, 33-35
Italy	2	32, 36
Indonesia	1	37

Table 2. Main subject domains of the studies

Study Domains	Total	%	References
Geometry	12	60	18, 20, 24-27, 29, 30, 33-35, 37
Algebra	1	5	28
Geometry, algebra, and calculus	1	5	32
Mathematical functions	2	10	19, 22
Other (basic mathematics and school mathematics)	4	20	21, 23, 31, 36

We then examined which curricula using AR technology were featured in the studies. Geometry was the most widely studied topic in mathematics (n = 12 articles), followed by basic/school mathematics (n = 4), while two articles mentioned mathematical functions as their main topic. Finally, one article was based on an algebra curriculum and one on a mixture of algebra, geometry, and calculus (Table 2).

Furthermore, we were interested in identifying the educational level of the participants in these studies. They were mostly middle-school students and teachers (n = 5 articles). Elementary-education students follow in four articles (n = 4), two of which concerned pupils with learning disabilities: students with special educational needs (SEN) and students identified through Malaysia's Literacy and Numeracy Screening program as requiring additional support due to physical, cognitive, or emotional challenges. Two studies focused on higher education (n = 2). In the final five articles, education level was not mentioned. The number of participants in the studies ranged from 5 to 82 (Table 3).

The next step was to examine the hardware used to implement AR in the studied educational contexts. Android tablets and personal computers were the most popular hardware tools reported,[18,19,21,22,24-26,28-31,33,34] with mobile phones[18,20-22,24,25,28,29,36,37] also seeing a high rate of use, which can be attributed to mobile devices' suitability for use in classroom environments and their accessibility, affordability, and portability (Table 4).

Of the methods used to evaluate the effectiveness of the use of AR technology in mathematics education, the most popular strategy adopted by the researchers was the questionnaire at a rate of 45% (n = 9 articles)[19,21-23,28,29,33,36,37] and pre-test/post-test at a rate of 40% (n = 8).[19,20-22,23,25,31,36] Interviews,[19,33] video recordings/transcriptions,[32,33] and content analysis[33,34] followed at a rate of 10% (n = 2), and 5% mentioned the use of observation cards (n = 1).[21] A further 20% of the reviewed studies did not mention the evaluation method (Table 5).[18,26,27,35]

The implementation of AR in the context of mathematics education resulted in various essential outcomes (Table 6). Regarding students' cognitive and metacognitive achievements, several studies have reported positive outcomes from engaging students in activities within an AR teaching environment. Conceptual understanding was mentioned as an important effect of AR use in educational settings in many of the reviewed studies.[18,19,22,23,27,28,30,31,33-35,37] Meanwhile, academic achievement[19,21-23,27,29,35] and the development of visual–spatial thinking[19,20,22,24-27,29,30,33-35] were also mentioned at a high rate, specifically in the field

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Table 3. Educational level and range of study samples

Educational level	Sample size	References
Secondary	82	19
Secondary	76	20
Secondary	24	32
Secondary	Not mentioned	30
Secondary	30	37
Primary	30	25
Primary	29	31
Primary (SEN)	22	23
Primary (LINIUS)	32	28
Undergraduate	48	22
Undergraduate	40	29
Teachers	10	29
Teachers	15	36
Teachers	36	21
Teachers	10	34
Teachers	5	33
Not mentioned	N/A	18, 24, 26, 27, 35

Abbreviations: SEN: Special educational needs; LINIUS: Literacy and Numeracy Screening.

Table 4. AR hardware used in reviewed studies

Hardware	Total	%	References
Android tablets and personal computers	13	65	18, 19, 22, 24-26, 28-31, 33-35
Camera and motion sensors	3	15	20, 21, 31
Marker-based systems	6	30	20, 21, 23, 24, 36
Mobile phones	10	50	18, 20-22, 24, 25, 28, 29, 36, 37
Checklists, guidelines, booklets, and magic books	6	30	20, 21, 23, 24, 36, 37
AR headsets/VR glasses	2	10	30, 32
Interactive whiteboard	1	5	32
Not mentioned	1	5	27

Abbreviation: AR: Augmented reality; VR: Virtual reality.

Table 5. Evaluation processes in the reviewed studies

Evaluation method	Total	%	References
Pre-test/post-test	8	40	19-23, 25, 31, 36
Questionnaires	9	45	19, 21-23, 28, 29, 33, 36, 37
Interviews	2	10	19, 33
Video recording/transcription	2	10	32, 33
Not mentioned	4	20	18, 26, 27, 35
Observation cards	1	5	21
Content analysis methods	2	10	33, 34

Table 6. Cognitive and metacognitive outcomes of AR use in mathematical education

Outcomes	Total	%	References
Academic achievement	7	35	18, 20-22, 26, 28, 34
Understanding	12	60	17, 18, 21, 22, 26, 27, 29, 30, 32-34, 36
Visuospatial thinking	11	55	18, 19, 21, 23-26, 28, 29, 32-34
Autonomy	5	25	23, 27, 29, 30, 33
Mathematical thinking/ reasoning, meaning-making, and problem-solving	11	55	22, 23, 25-27, 31-36
Critical thinking	2	10	23, 35
Creativity	1	5	34
Modeling skills	3	15	32-34
Transdisciplinary learning	4	20	17, 32-34

of geometry. In addition, significant enhancements were observed in students' mathematical thinking/reasoning, meaning-making, and problem-solving abilities.[23,24,26-28,32-37] Furthermore, five studies noted a progressive acquisition of autonomy,[24,28,30,31,34] and two highlighted the development of critical thinking for logical decision-making.[24,36] Creativity was mentioned in one study,[35] and another study discussed modeling skills achievement.[34,35] Finally, transdisciplinary learning was mentioned in four studies.[18,33-35]

In terms of socioemotional outcomes, most of the studies indicated that integrating AR into teaching and engaging students in AR activities led to increased motivation,[19,20,22-24,27-31,34,37] improved collaboration and teamwork among students in problem-solving activities,[24,26,27,31,35,37] and enhanced interest in learning, which made the educational experience more engaging.[22,23,28,30,37] In addition, two studies discussed students' interest in engaging in more AR-based learning opportunities in the future,[19,23] and another study highlighted the positive impact of AR activities in reducing students' anxiety about mathematics.[27] Finally, one study suggested that conducting AR activities at home with parents may strengthen family bonds and promote self-directed learning (Table 7).[19]

In terms of pedagogical outcomes, participants found AR activities to be useful[18,19,21,23,36] and easy to use[22,28,36,37] and found that they facilitated learning,[19,21-23,28,29,37] saved time,[28] and promoted active learning (Table 8).[27,37]

Based on the above findings, first, AR technology was found to significantly enhance students' visual thinking and spatial visualization skills. Studies such as those by Elsayed and Al-Najrani,[20] Flores-Bascuñana *et al.*,[25] and

Table 7. Socioemotional outcomes of AR use in mathematical education

Outcomes	N	%	References
Learning interest	5	25	22, 23, 28, 30, 37
Motivation, perseverance, ambition, enjoyment, and satisfaction	10	50	19, 20, 22-24, 27-31, 34, 37
Collaboration	8	40	21, 24, 26, 27, 31, 35-37
Anxiety/stress reduction	1	5	27
Interest in future AR use	2	10	19, 23
Self-learning/family bonding	1	5	19

Table 8. Pedagogical outcomes of AR as a learning tool in mathematical education.

Pedagogical outcomes	Total	%	References
Utility	4	20	18, 19, 21, 23, 36
Easy to use	4	20	22, 28, 36, 37
Facilitate learning	6	30	19, 21, 23, 28, 29, 37
Save time	1	5	28
Active learning	2	10	27, 37

Salinas *et al.*[26,27] highlighted how AR-based instruction improved students' ability to grasp 3D geometric concepts and visualize abstract ideas.

Second, AR was found to enhance academic motivation and engagement. Elsayed and Al-Najrani,[20] Salinas,[26,27] and Awang *et al.*[28] found that learners who used AR applications were more motivated and engaged in mathematics than those who used standard teaching methods. The interactive and immersive nature of AR fostered a positive learning environment that encouraged active participation and exploration.

Third, AR promoted problem-solving and critical thinking. Vakaliuk *et al.*,[30] Nindiasari *et al.*,[37] and Li *et al.*[19] demonstrated that AR enhances students' ability to solve mathematical problems, particularly in geometry and spatial reasoning. A hands-on approach to learning through AR encouraged students to apply their theoretical knowledge in real-world scenarios, further developing their problem-solving abilities.

In addition, AR facilitated interdisciplinary learning. El Bedewy *et al.*[33-35] and Botana *et al.*[18] explored how AR can connect mathematics with cultural, historical, and architectural contexts. By modeling historical structures and integrating automated reasoning in mathematical explorations, AR deepened students' comprehension and bridged abstract concepts with real-world applications.

Furthermore, AR technology was noted to be accessible and cost-effective. Fernández-Enríquez and Delgado-Martín[24] emphasized that AR applications can run on various devices, making them feasible for classrooms with diverse technological resources. Moreover, the affordability of AR teaching materials made them an attractive choice for educators seeking innovative yet practical educational solutions.

Despite the similarities in the findings, the studies varied in their specific focus areas and methodologies. Most of the studies employed a quasi-experimental design, involving experimental and control groups to measure the effectiveness of AR in improving mathematical skills. For instance, Elsayed's research on middle-school students in Saudi Arabia and Lozada-Yán's study on Ecuadorian third graders utilized pre- and post-tests to compare outcomes.[20,31] This design ensured a structured evaluation of AR's impact on learning.[24,26-28,37] While Flores-Bascuñana's work on the teaching of 3D geometric concepts in a primary-school class did not include a pre-intervention evaluation, limiting the conclusiveness of the results,[25] Alibraheim's study on teacher training incorporated a rigorous pre-/post-test design to assess skill enhancement in using AR applications.[21]

The technological tools and platforms used in the studies also differed. While Fernández-Enríquez and Delgado-Martín utilized Unity along with Vuforia to create AR resources,[24] Kounlaxay *et al.* and Del Cerro Velázquez and Morales Mendez leveraged GeoGebra AR to teach geometric concepts.[22,29] These differences highlight the versatility of AR tools in various educational contexts.

Moreover, some studies extended the applications of AR beyond traditional classroom settings. For example, Cascales-Martínez *et al.* explored the use of a multi-touch tabletop system for teaching money management to students with SEN,[23] and El Bedewy *et al.* examined AR's role in museum-based STEAM education, connecting mathematical learning with cultural and historical contexts.[33-35]

2.2.2. Challenges of employing AR technology in mathematics curricula

Only seven studies identified issues with AR technologies that could impact the effectiveness of learning activities (Table 9). Two-thirds of the students participating in one intervention[19] provided valuable feedback for app improvement and expressed willingness to participate in more AR-based learning experiences. Most of the learners suggested updates to tablet computer configurations, stable card recognition systems, realistic 3D scene simulations, diverse real-life and game contexts, and the time allocated for play and problem-solving. Interview results confirmed these recommendations; in addition, the interviews

Table 9. Challenges and recommendations for AR-supported education

Issue	Description	Recommendations	References
Autonomy, concentration, and confidence	Low-achieving learners struggled with these aspects and needed teacher support	Provide teacher training to foster encouragement and engagement among students during AR activities	23, 28, 29, 31
Teachers' challenges in implementing AR	Teachers struggled with integrating AR in their lessons	Implement comprehensive teacher training programs that focus on technical AR skills and pedagogy	24, 29
Technical limitations	Problems with tablet configurations, card recognition, and 3D scene simulations	Improve app designs with better device configurations, realistic simulations, and stable recognition systems	19
Lack of prior knowledge assessment	Failure to assess students' knowledge before AR activities	Include prior knowledge assessments to provide tailored AR interventions	25
Limited availability of AR in schools	Lack of devices such as tablets, smartphones, and high-speed internet in under-resourced schools	Promote low-cost or free AR apps, seek partnerships for device donations, implement shared-device models, and integrate collaborative learning models	24
Visual issues	Students requested enhanced visuals, such as better graph colors and font sizes	Optimize the visual design of AR apps to improve clarity and usability	19
Lack of AR tools	AR tools are not widely available in less developed regions	Encourage the use of open-source platforms such as GeoGebra and mobile apps with AR functionality	29
Unaffordable AR platforms	High cost of AR platforms and devices	Develop more affordable AR solutions and integrate them into educational curricula	19

identified improvements such as optimizing visual effects on the screen including graph colors and font sizes. Notably, these students explored the game independently due to the game-based design of the AR app, which lacked social interactivity.

Low-achieving learners, who comprised most of those reporting issues in AR learning environments, expressed concerns about autonomy, concentration, and self-confidence.[23,28,29,31] They often required teachers' reassurance and encouragement from teachers to engage or persist in AR activities. Furthermore, only two studies addressed teachers' challenges in implementing AR activities.[24,29] Finally, Flores *et al.* reported that their results lacked strength as they did not include an evaluation of learners' prior knowledge.[25]

Despite its educational potential, a key limitation of AR technology, as highlighted in the study by Fernández-Enríquez and Delgado-Martín[24] (2020), was its limited availability, particularly in under-resourced educational settings. Many schools and students lacked access to necessary devices such as smartphones, tablets, or high-speed internet, creating barriers for the widespread adoption of AR in classrooms.

To address these challenges, the study suggested adopting low-cost AR solutions, many of which are available as free or affordable apps compatible with basic mobile devices, which will reduce the need for expensive hardware. The authors also recommended that schools and technology providers form partnerships to secure funding or device donations to bridge the digital divide. They emphasized the importance of teacher training programs to ensure that educators can effectively integrate AR into lessons, covering both technical proficiency and pedagogical strategies that will enhance students' learning of complex concepts. Teachers should be trained to develop AR-based learning materials and use them to improve students' spatial reasoning skills. In addition, the study proposed incorporating AR into collaborative learning environments, where students share devices, thus promoting teamwork and ensuring that more students benefit from the technology without needing one-to-one device availability. This model not only improves access but also fosters peer learning and collaboration.[24]

A similar issue with AR's limited availability was noted in the study by Kounlaxay *et al.*,[29] particularly regarding less developed regions. They suggested adopting open-source tools like GeoGebra and mobile applications with AR functionality. Teacher training was again emphasized as a critical component to overcoming technological constraints, helping to make AR-supported education more accessible and effective.[29] Li *et al.*[19] further recommended developing more affordable AR platforms and integrating them into curricula to complement traditional teaching methods.[19]

2.2.3. Connecting humanities and mathematics through AR

Of the 20 reviewed studies, four studies explored the integration of arts, architecture, history, and culture into mathematics education using AR technology. Among these, two studies specifically emphasized non-formal learning environments, such as museums and outdoor settings. For example, in 2024, El Bedewy *et al.*[33] investigated the role of museums as facilitators of interdisciplinary learning experiences. The researchers investigated various museum activities and programs that connected mathematics with arts, architecture, culture, and history, such as learners using AR to explore mathematical patterns in architectural structures (e.g., Islamic geometric designs and historical buildings) and overlay mathematical grids onto exhibits such as sculptures and paintings. This enabled students to learn about mathematical concepts such as symmetry, tessellation, scaling, and proportions in a real-world cultural context. The impact on students' learning outcomes was significant, as AR increased engagement, improved comprehension of abstract mathematical ideas, and enhanced problem-solving skills by connecting mathematics with culturally relevant artifacts. This analysis highlighted the potential of museums as conducive environments for STEAM education, offering diverse opportunities for interdisciplinary learning and exploration. The museum served as a space where learners could engage in transdisciplinary practices that foster creativity and modeling skills by interacting with historical and cultural artifacts. The museum in this context functioned as more than a traditional exhibition space; it became a space where students and educators could explore these disciplines in a hands-on manner, using technology such as GeoGebra for architectural modeling and learn about the visual arts and cultural history. This approach allows learners to deepen their understanding of mathematical concepts and their connections to the broader world.[33]

Meanwhile, the initiative by Botana *et al.*[18] was centered on leveraging AR technology to enhance outdoor learning experiences with mathematics. By automatically generating AR content related to mathematical concepts and embedding it into outdoor settings, the researchers provided students with engaging and immersive learning opportunities. This approach sought to increase student engagement, foster a deeper understanding of mathematical concepts, and promote outdoor learning. The examples provided, including the {8/2} polygon at Sardinero Beach and Okuda's artwork *Infinite Eye I*, illustrated practical applications of mathematical concepts in real-world scenarios. These examples demonstrated the potential of technology such as GeoGebra in helping students validate mathematical principles and analyze artistic creations within AR-enhanced learning environments.[18]

Earlier studies by El Bedewy *et al.* from 2021[34] and 2022[34] also proposed innovative approaches for utilizing the arts as pedagogical tools for teaching mathematical concepts. El Bedewy *et al.*[34] demonstrated how AR could be employed to integrate art, culture, and architecture into mathematics education. AR enabled participating students to explore historical structures, such as temples and bridges, and analyze the embedded mathematical principles in their designs, including symmetry, proportion, and geometric patterns. In addition, AR was used to project visual representations of culturally significant artworks, allowing students to examine mathematical concepts such as scaling, perspective, and tessellation. This approach had a substantial impact on students' learning, enhancing their engagement, problem-solving skills, and conceptual understanding, particularly in geometry and spatial reasoning. AR also fostered creativity and critical thinking by connecting mathematics to broader cultural and historical contexts.[34]

The 2021 study[34] entailed the use of architectural models, such as Cheomseongdae and Dendera Temple, to enhance mathematical understanding. Students analyzed Cheomseongdae mathematically using the Surface of Revolution concept before creating 3D models in GeoGebra. Deviation from the prescribed steps prompted students to create alternative models, encouraging creativity and exploration. Similarly, the Dendera Temple model requires basic geometric skills and an understanding of shape relationships. Students reconstructed the temple using simple shapes, exploring connections between length, height, and width. Teachers could prompt students to either imitate existing models or innovate on them, fostering problem-solving and critical thinking skills. This approach allowed students to visualize designs in AR and produce physical copies through 3D printing, facilitating deeper comprehension without physical access to architectural sites. Overall, this educational method promoted creativity, collaboration, and critical thinking among students, enriching their understanding of mathematical and architectural concepts[35] Table 10 provides insight into the approaches used to integrate mathematics with arts, architecture, history, and culture in educational settings.

3. Discussion

This systematic review investigated the evolving role of AR in mathematics education, the results achieved, the challenges faced, and the integration of AR with art to enhance students' learning.

3.1. Applications and outcomes of using AR in mathematics education

AR was applied in mathematics education through various means, including integration with specific mathematical

Table 10. Arts in mathematics education through AR

References	Architecture	Arts	Culture	History	Technologies	Learning skills	Educational environment
33	✓	✓	✓	✓	AR, GeoGebra 2D/3D, 3D printing	Modeling, visualization, understanding	Indoor museums
35	✓	✓	✓	✓	AR, 3D printing	Modeling, problem-solving, creative thinking, visualization	Indoor classroom
34	✓	-	✓	✓	AR/VR, GeoGebra 2D/3D, 3D printing/scanning, origami, 4D frames	Modeling, problem-solving, critical thinking, visualization, understanding	Indoor classroom, outdoors, online, museums
18	✓	✓	-	-	AR, GeoGebra 2D/3D	Engagement, understanding, immersive outdoor learning, automated reasoning	Outdoor educational settings

topics such as geometry.[38] A notable application was modeling,[39] in which students could create virtual 3D objects on real-world surfaces.[40] They could measure and label objects to determine their real values, facilitating learning such as calculating the volume of a prism. Another application was geometry, particularly in developing spatial abilities[41] and recognizing the properties of different shapes.[42] AR applications facilitated the visualization of abstract mathematical concepts, aiding in the development of geometric thinking.[43] Moreover, they allowed students to examine geometric bodies and their properties in detail, aiding the development of spatial skills. Educators designed AR-based learning environments using mobile applications, tabletop systems, and AR-enhanced field trips to provide students with interactive and immersive learning experiences.

Moreover, AR applications were found to enhance the learning process, motivation, and efficiency. These applications improved student performance and increased motivation by enabling new learning experiences, saving time, enhancing lab skills and attitudes, and fostering critical thinking, problem-solving, and communication skills.[9-11] Students demonstrated improved comprehension and academic performance in mathematics due to the enhanced visualization of mathematical concepts and real-world applications afforded by AR technology.[12] Furthermore, students who engaged in AR-enhanced learning experiences[44] developed their socioemotional skills and cognitive abilities. AR-enhanced field trips to museums,[45] architectural sites, or natural landmarks provided students with interactive learning experiences and additional information.[13] In addition, the incorporation of game elements and challenges into mathematics education using AR was shown to increase student engagement and enjoyment.[14] Increased motivation and engagement were overall the most striking changes, as students found AR-based activities more engaging and enjoyable than traditional instructional methods.[4,44]

3.2. Challenges of employing AR technology in mathematics education

Although AR offered numerous benefits to mathematics education, its implementation presented several challenges. Technical constraints, such as the reliability and accessibility of the AR technology, presented significant obstacles for educators. Researchers delineated various limitations of educational AR applications, encompassing usability issues, student distraction, and technical hurdles.[19,23-25,28,29,31] While some technical issues can be resolved with time, these investigations demonstrated that challenges such as inadequate teacher training and lacking social acceptance persist, hindering the widespread adoption and effectiveness of AR in education and society.[8] To overcome these challenges, comprehensive teacher training is paramount, which would enable educators to effectively integrate AR into their teaching practices while addressing the associated usability concerns and practical limitations. The slow diffusion of AR technology in educational settings emphasizes the necessity for heightened awareness and acceptance among educators and stakeholders.[16]

3.3. Combining AR technology and mathematics with the arts

The traditional education system often imposes rigid boundaries between the arts and the sciences, hindering holistic development. However, historical figures such as Leonardo Da Vinci have proved the value of integrating the arts and sciences for innovative problem-solving. "Active learning," which blends STEM subjects with the arts, offers a more engaging educational approach.[15] Aligning with the STEAM framework, the combination of AR with arts and culture in mathematics education promotes holistic learning[46] and nurtures student creativity.[47,48] Assignments that integrate works of art or architectural monuments with mathematical concepts reinforce the interdisciplinary nature of education.[49] By infusing the arts into STEM subjects, AR technology enhances collaboration and engagement in STEAM education, providing students with interdisciplinary learning opportunities.[50] Through AR-enhanced activities, students can explore the intricate connections between mathematics and the arts in field such as architecture, gaining a profound appreciation for the practical applications of mathematical concepts in real-world contexts.[51]

4. Conclusions

Looking ahead, we believe that this study's findings will prove useful to educators and course designers of mathematics curricula. The integration of the arts in mathematics education through AR also presents a promising avenue for future research. That said, the challenges mentioned above suggest the need for continued investigation and innovation to enhance the effectiveness of AR in mathematics education. Furthermore, machine learning and its applications in visualizing and detecting new, hidden geometric features in art objects (e.g., drawings, paintings, and monuments) for automated reasoning in tools such as GeoGebra is a potential avenue for future work.

Our reliance on the Scopus database presents a limitation, as it is likely that relevant studies from other electronic sources were missed. Expanding the search to additional databases would likely have strengthened the review. However, we chose Scopus as the sole database due to its comprehensive coverage across various disciplines and its access to high-quality, peer-reviewed journals. The advanced search capabilities and robust citation tracking within Scopus allowed for precise, efficient literature retrieval. Using a single, reputable database streamlined the research process and ensured consistency and reliability, which are critical for the reproducibility of the findings. In addition, Scopus's widespread availability through academic institutions made it a convenient and dependable choice.

This literature review provides a comprehensive overview of the evolving role of AR in mathematics education, particularly its integration with the arts and culture. Analyzing 20 English-language journal articles from the Scopus database following the PRISMA guidelines, we revealed AR's effectiveness in enhancing students' visual thinking, spatial visualization, and comprehension of 3D geometric concepts in mathematics education, with geometry being the most extensively studied topic. The findings emphasize AR's significant impact on fostering student motivation and engagement, as its interactive and immersive nature creates a positive learning environment that facilitates active participation and exploration. This leads to improvements in students' problem-solving skills and critical thinking, with quasi-experimental studies consistently showing the benefits of AR in boosting academic performance.

In addition, AR facilitates interdisciplinary learning by connecting mathematics with real-world cultural, historical, and architectural contexts, broadening students' understanding and providing applications that made learning more relevant and engaging for them. Examples included AR-enhanced field trips and projects that integrated mathematics with architecture, the arts, and history, demonstrating AR's potential to foster creativity, collaboration, and critical thinking. Notably, however, the review also highlights pertinent challenges, such as technical constraints and a lack of comprehensive teacher training, that hinder effective implementation. Future research should address these issues by exploring diverse AR applications, conducting rigorous evaluations, and developing accessible and cost-effective AR tools. Emphasizing the integration of AR with the arts will offer valuable insights into leveraging interdisciplinary approaches to enhance mathematics education. This study's findings highlight AR's transformative potential in mathematics education, promising a future full of dynamic, interactive, and engaging learning environments.

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